Predicting the Temporal Limits of Gaussianity in the Saturn-Enceladus System with the Unscented Transform

Benjamin L. Hanson
Ph.D. Student, Jacobs School of Engineering
Department of Mechanical and Aerospace Engineering
UC San Diego, La Jolla, CA

Dr. Aaron J. Rosengren
Assistant Professor, Jacobs School of Engineering
Department of Mechanical and Aerospace Engineering
UC San Diego, La Jolla, CA

Dr. Thomas R. Bewley
Professor, Jacobs School of Engineering
Department of Mechanical and Aerospace Engineering
UC San Diego, La Jolla, CA

Dr. Todd A. Ely
Principal Navigation Engineer
Jet Propulsion Laboratory, California Institute of Technology
Pasadena, CA

The Nonlinear State Estimation Problem

- Consider the state estimation of a general system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t) + \boldsymbol{w}, \quad \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, t) + \boldsymbol{v}$$

where $\boldsymbol{x} \in \mathbb{R}^d$ is a realization of random variable $\boldsymbol{X}$

- If $\boldsymbol{f}, \boldsymbol{h}$ are linear and $\boldsymbol{w}, \boldsymbol{v}$ are Gaussian zero-mean white noise, then

$$\boldsymbol{X}(t) \sim \mathcal{N}\left(\boldsymbol{\mu}(t), \boldsymbol{\Sigma}(t)\right) = \frac{1}{\sqrt{(2\pi)^d |\boldsymbol{\Sigma}(t)|}} \exp\left(-\frac{1}{2}\left(\boldsymbol{x} - \boldsymbol{\mu}(t)\right)^T \boldsymbol{\Sigma}(t)^{-1}\left(\boldsymbol{x} - \boldsymbol{\mu}(t)\right)\right)$$

- However, if $\boldsymbol{f}, \boldsymbol{h}$ are nonlinear, then generally speaking

$$\boldsymbol{X}(t) \sim p(\boldsymbol{x}, t) \neq \mathcal{N}\left(\boldsymbol{\mu}(t), \boldsymbol{\Sigma}(t)\right)$$

Fundamental Questions

1. How do we measure Gaussianity?
2. How long does it take for state uncertainty to become non-Gaussian?
3. Can we predict when state uncertainty is becoming non-Gaussian with an abstraction more efficient to propagate than a dense Monte Carlo?

Being "kind-of" Gaussian

Analytical vs. Statistical Definitions

> "Being 'kind-of' Gaussian is like being 'kind-of' dead."
>
> *-Dr. Tom Bewley, UCSD*

Analytical Definition of a Gaussian

$$p(\boldsymbol{x}\,|\,\boldsymbol{\mu};\boldsymbol{\Sigma}) = \frac{1}{\sqrt{(2\pi)^d\,|\,\boldsymbol{\Sigma}\,|}}\exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right)$$

Statistical Definition of a Gaussian

A Monte Carlo comparison of the Type I and Type II error rates of tests of multivariate normality

CHRISTOPHER J. MECKLIN†* and DANIEL J. MUNDFROM‡

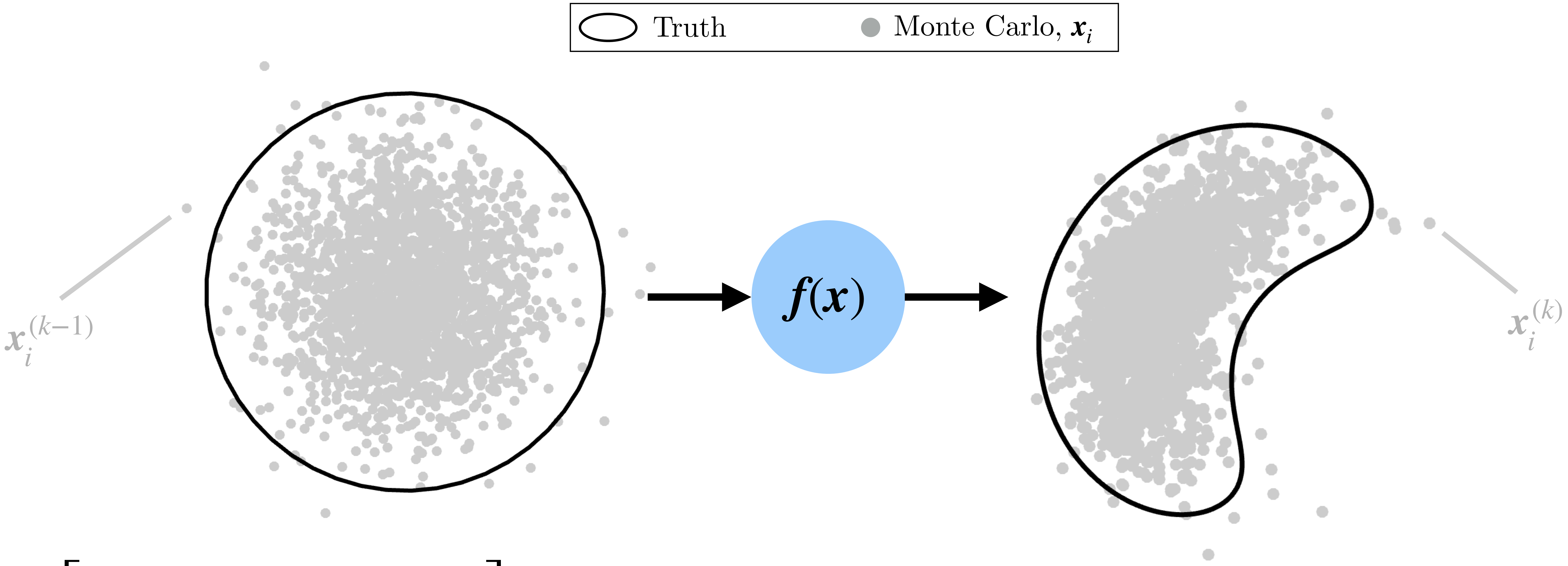
†Department of Mathematics and Statistics, Murray State University, Murray, KY 42071, USA
‡Department of Applied Statistics and Research Methods, University of Northern Colorado, CO, USA

Table 1. Tests of MVN.

Test	Class	Iris setosa
Mardia's skewness	Skewness/kurtosis	Do not reject
Mardia's kurtosis	Skewness/kurtosis	Do not reject
Hawkins	Goodness-of-fit	Reject
Koziol	Goodness-of-fit	Do not reject
Mardia–Foster	Skewness/kurtosis	Reject
Royston	Goodness-of-fit	Reject
PRS	Goodness-of-fit	Do not reject
Henze–Zirkler	Consistent	Do not reject
Mardia–Kent	Skewness/kurtosis	Do not reject
Romeu–Ozturk	Goodness-of-fit	Reject
Singh (classical)	Graphical/Correlational	Reject
Singh (robust)	Graphical/Correlational	Reject
MSL	Goodness-of-fit	Do not reject

Henze-Zirkler Statistic

$d = 2$
$n = 2000$



$$\mathrm{HZ} = \left[\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\exp\left(-\frac{\beta^2}{2}D_{ij}\right)\right] - \left[2\left(1+\beta^2\right)^{-\frac{d}{2}}\sum_{i=1}^{n}\exp\left(-\frac{\beta^2}{2(1+\beta^2)}D_i\right)\right] + \left[n(1+2\beta^2)^{-\frac{d}{2}}\right]$$

- d = dimensionality
- n = # of Monte Carlo samples
- $\beta = \frac{1}{\sqrt{2}}\left(\frac{n(2d+1)}{4}\right)^{\frac{1}{d+4}}$, smoothing parameter
- $D_{ij} = \left(\boldsymbol{x}_i^{(k)} - \boldsymbol{x}_j^{(k)}\right)^T \Sigma^{(k)^{-1}} \left(\boldsymbol{x}_i^{(k)} - \boldsymbol{x}_j^{(k)}\right)$, Mahalanobis distance between each point and every other point
- $D_i = \left(\boldsymbol{x}_i^{(k)} - \boldsymbol{\mu}^{(k)}\right)^T \Sigma^{(k)^{-1}} \left(\boldsymbol{x}_i^{(k)} - \boldsymbol{\mu}^{(k)}\right)$, Mahalanobis distance between each point and the mean

$\mathrm{HZ} > \mathrm{HZ}^*(\alpha = 0.003) \Rightarrow H_0$ should be rejected
$\mathrm{HZ} \leq \mathrm{HZ}^*(\alpha = 0.003) \Rightarrow H_0$ cannot be rejected

- HZ is approximately log-normally distributed, so a null hypothesis H_0 of Gaussianity may be tested

Henze-Zirkler Statistic

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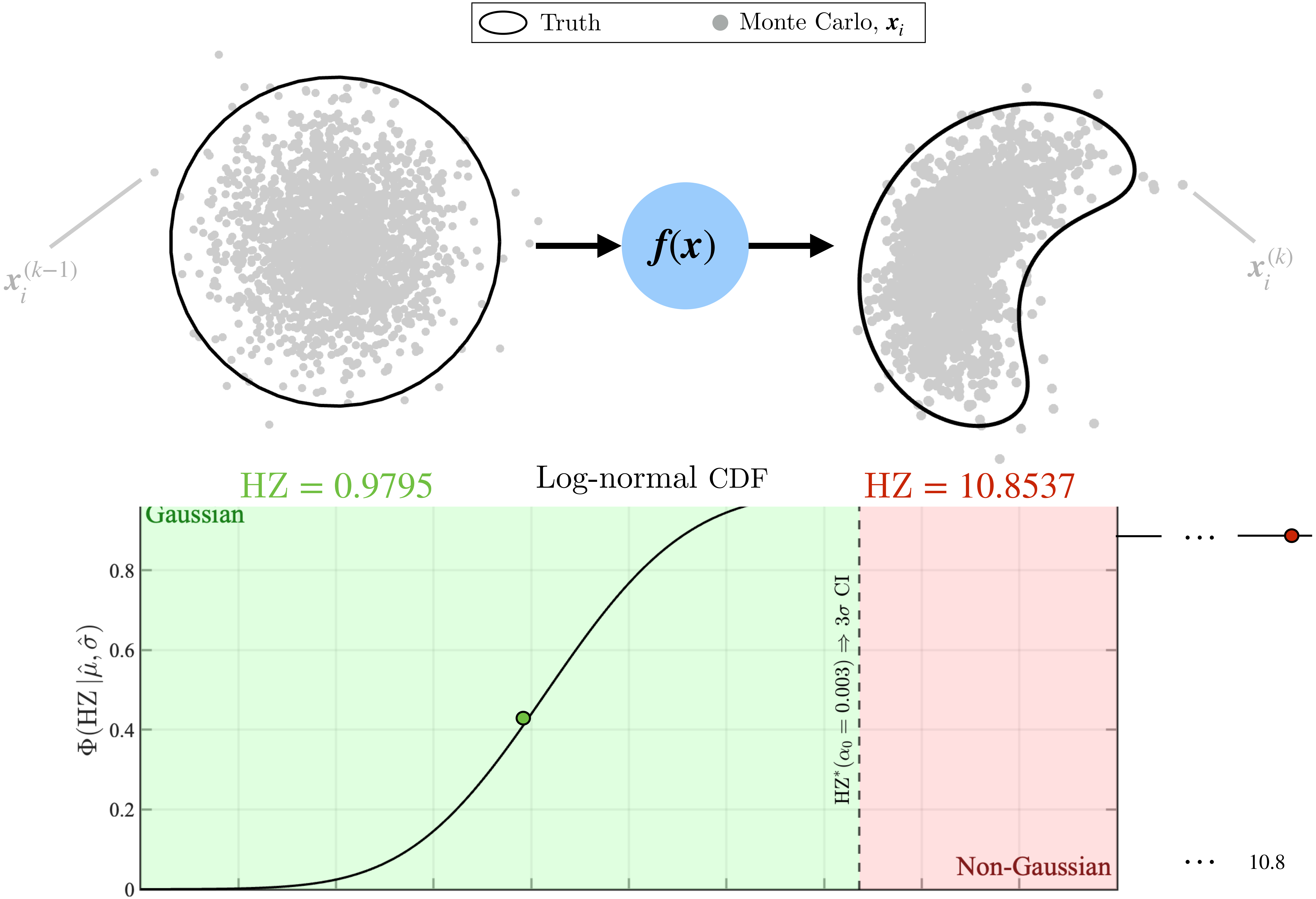
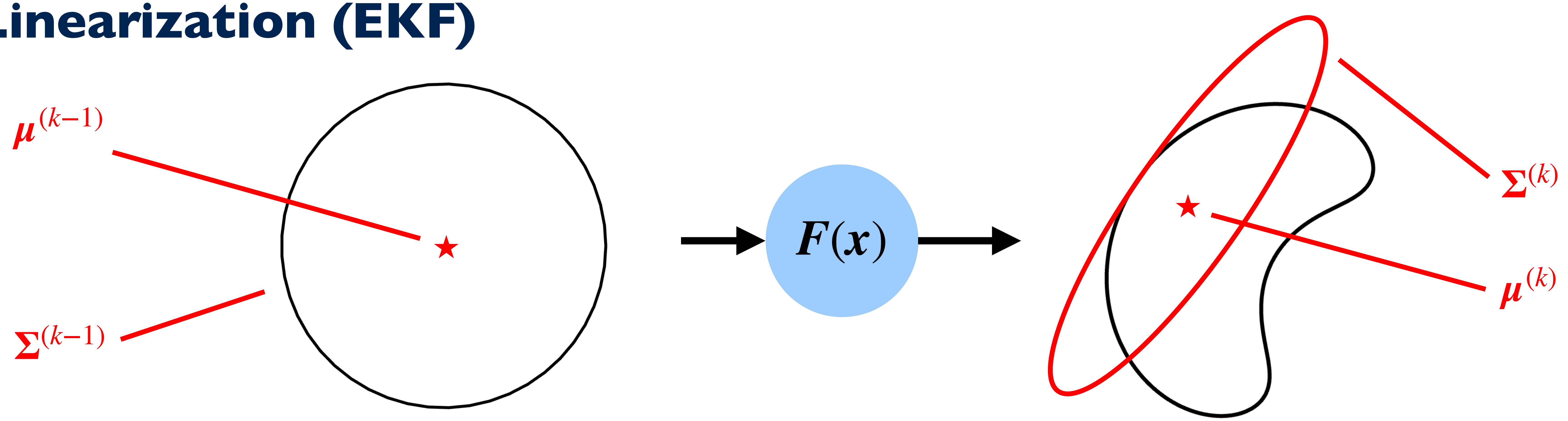


The Unscented Transform

"...it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function..."

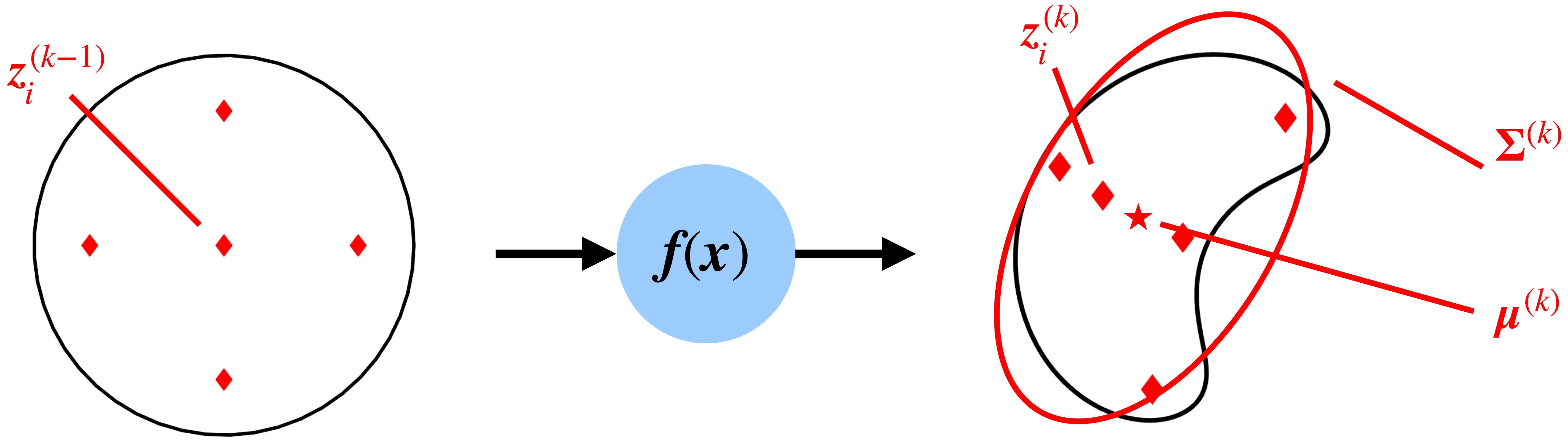
-Dr. Jeffrey Uhlmann, Inventor of the Unscented Transform

Analytical Linearization (EKF)

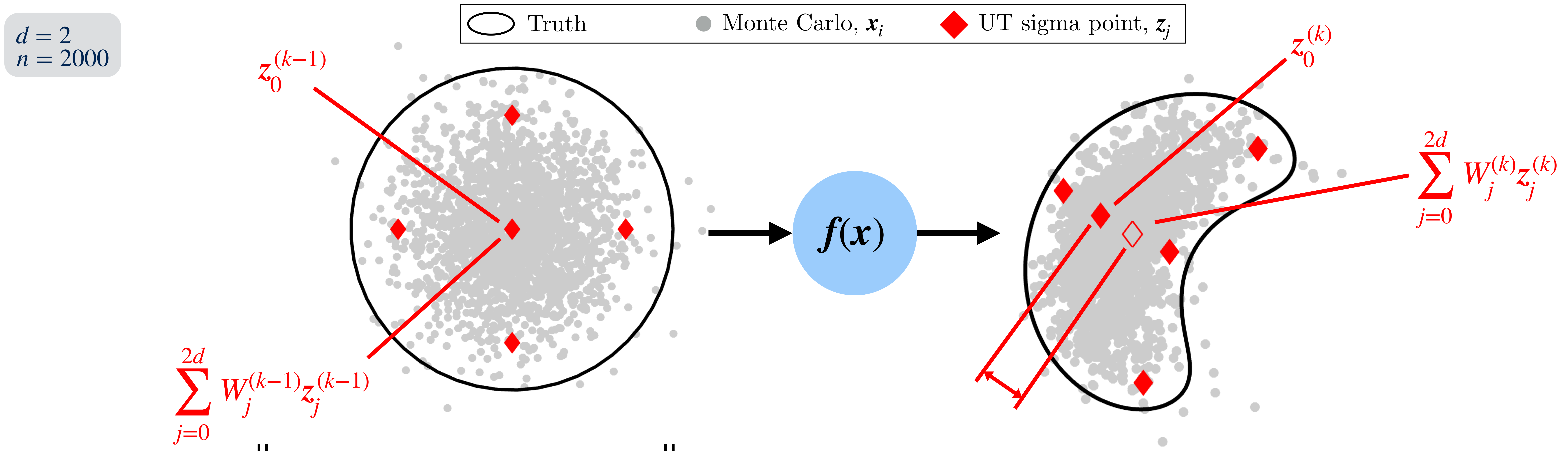


Statistical Linearization (UKF)

States	Weights
$z_0^{(k-1)} = \boldsymbol{\mu}^{(k-1)}$	$W_0^{(k-1)} = \kappa/(d+\kappa)$
$z_i^{(k-1)} = \boldsymbol{\mu}^{(k-1)} + \left(\sqrt{(d+\kappa)\Sigma^{(k-1)}}\right)_i$	$W_i^{(k-1)} = \kappa/\left(2(d+\kappa)\right)$
$z_{i+d}^{(k-1)} = \boldsymbol{\mu}^{(k-1)} - \left(\sqrt{(d+\kappa)\Sigma^{(k-1)}}\right)_i$	$W_{i+d}^{(k-1)} = \kappa/\left(2(d+\kappa)\right)$



Normalized Euclidean Distance



$$\text{NED} = \left\| L^{-1} \left(z_0^{(k)} - \sum_{j=0}^{2d} W_j^{(k)} z_j^{(k)} \right) \right\| ,$$ where L^{-1} is the inverse lower triangular of the covariance

- NED may be calculated from the UT sigma points alone, meaning it requires a fraction of the samples that the HZ requires for an accurate value
- When $\boldsymbol{f}(\boldsymbol{x})$ is linear, NED remains at 0; when $\boldsymbol{f}(\boldsymbol{x})$ is nonlinear, NED may drift

Normalized Euclidean Distance

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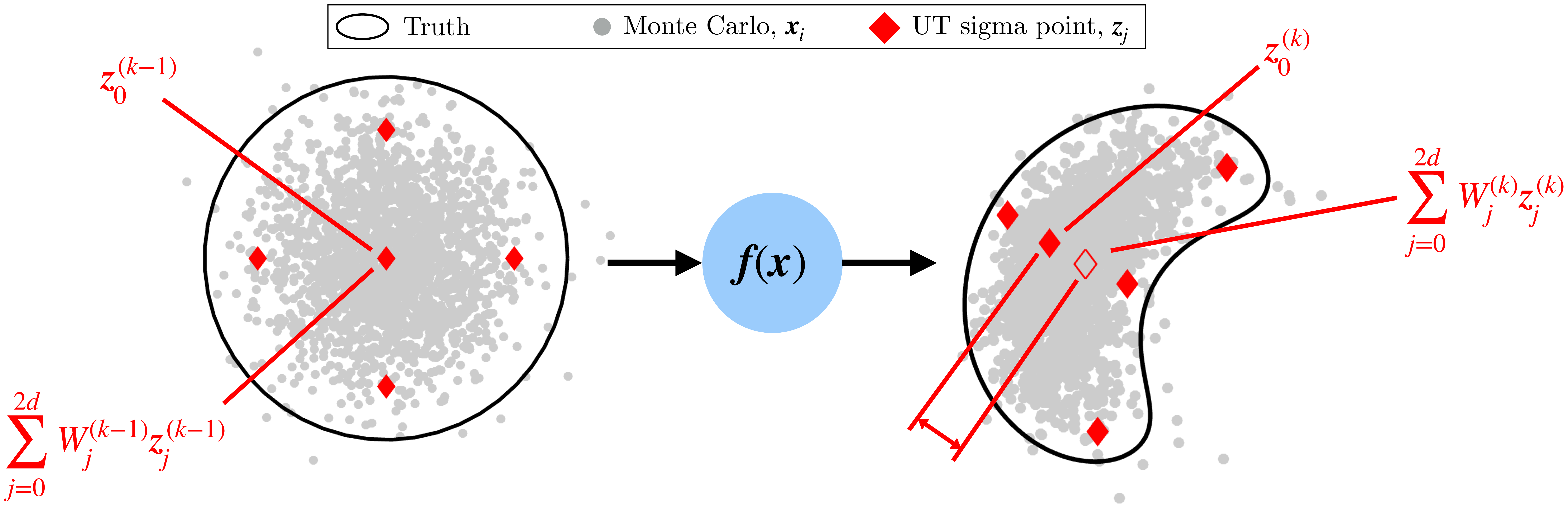
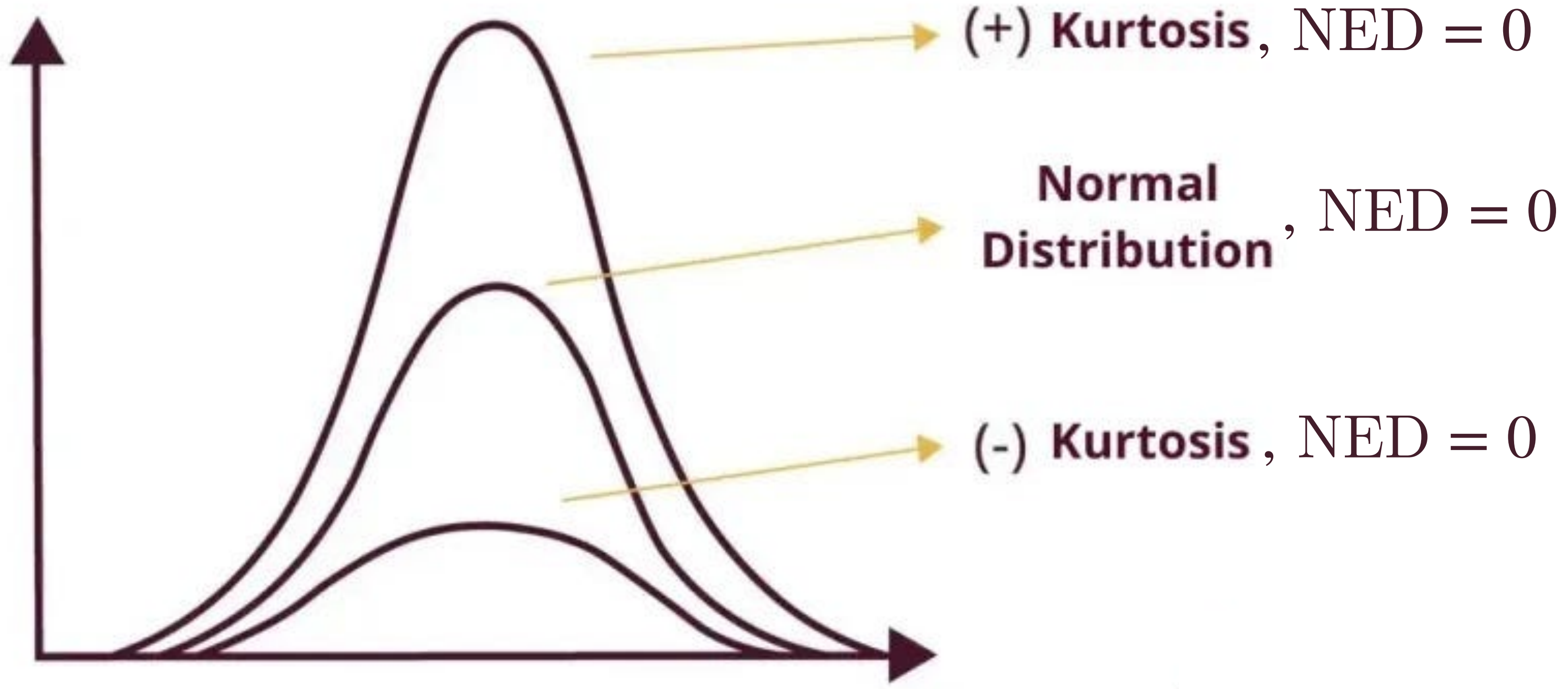
$n = 2000$



NED = 0 NED = 0.1691

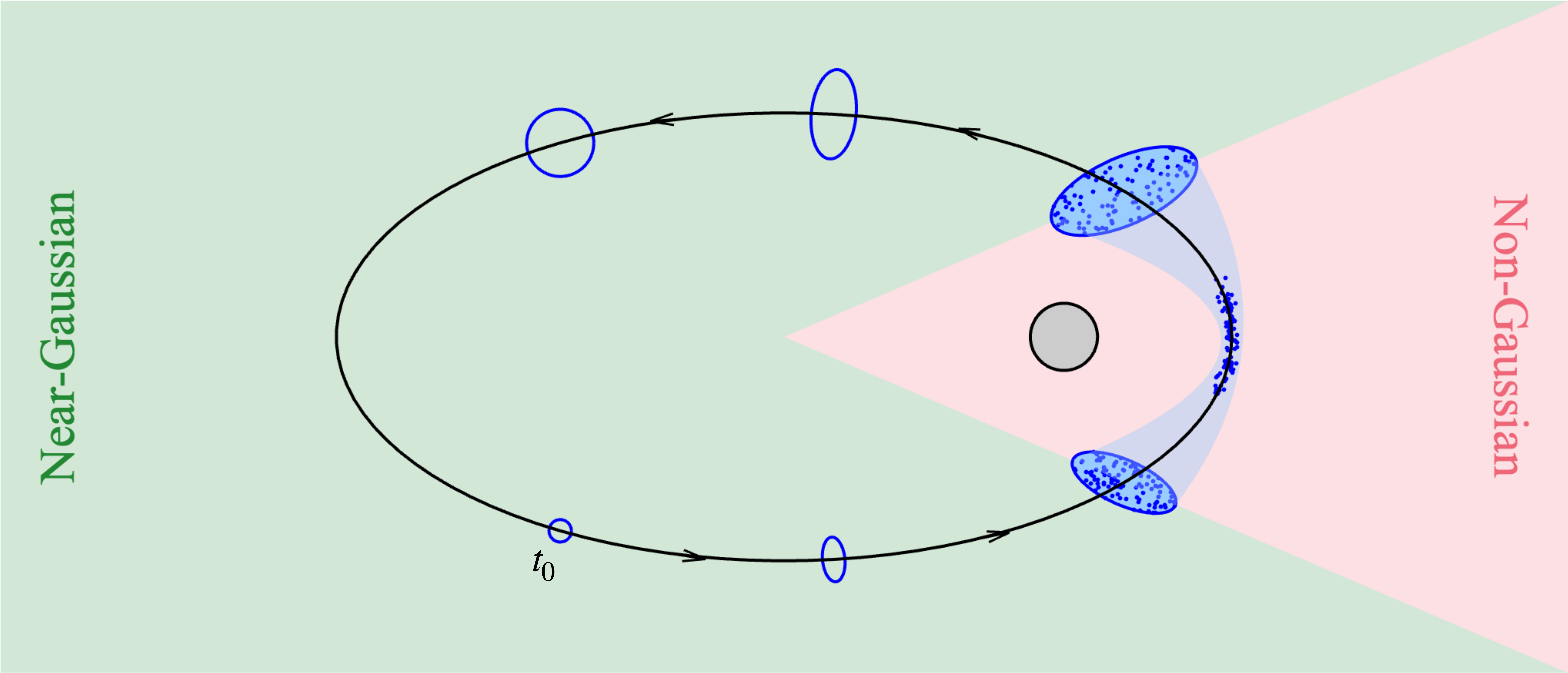
- NED is not a consistent statistical test; without mapping it to a consistent statistical test (HZ) we have no absolute information on the likelihood that the sigma points come from a Gaussian distribution

- **What about kurtosis?**



Two-Body Problem HZ-NED Mapping

- State uncertainty in closed orbits tends to oscillate between near-Gaussian during the quiescent, rectilinear phases and highly non-Gaussian near periapsis



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Check for updates

State Uncertainty Normality Detection

Introducing an Unscented Transform-Based Test

Sven K. Flegel[1] · James C. Bennett[2]

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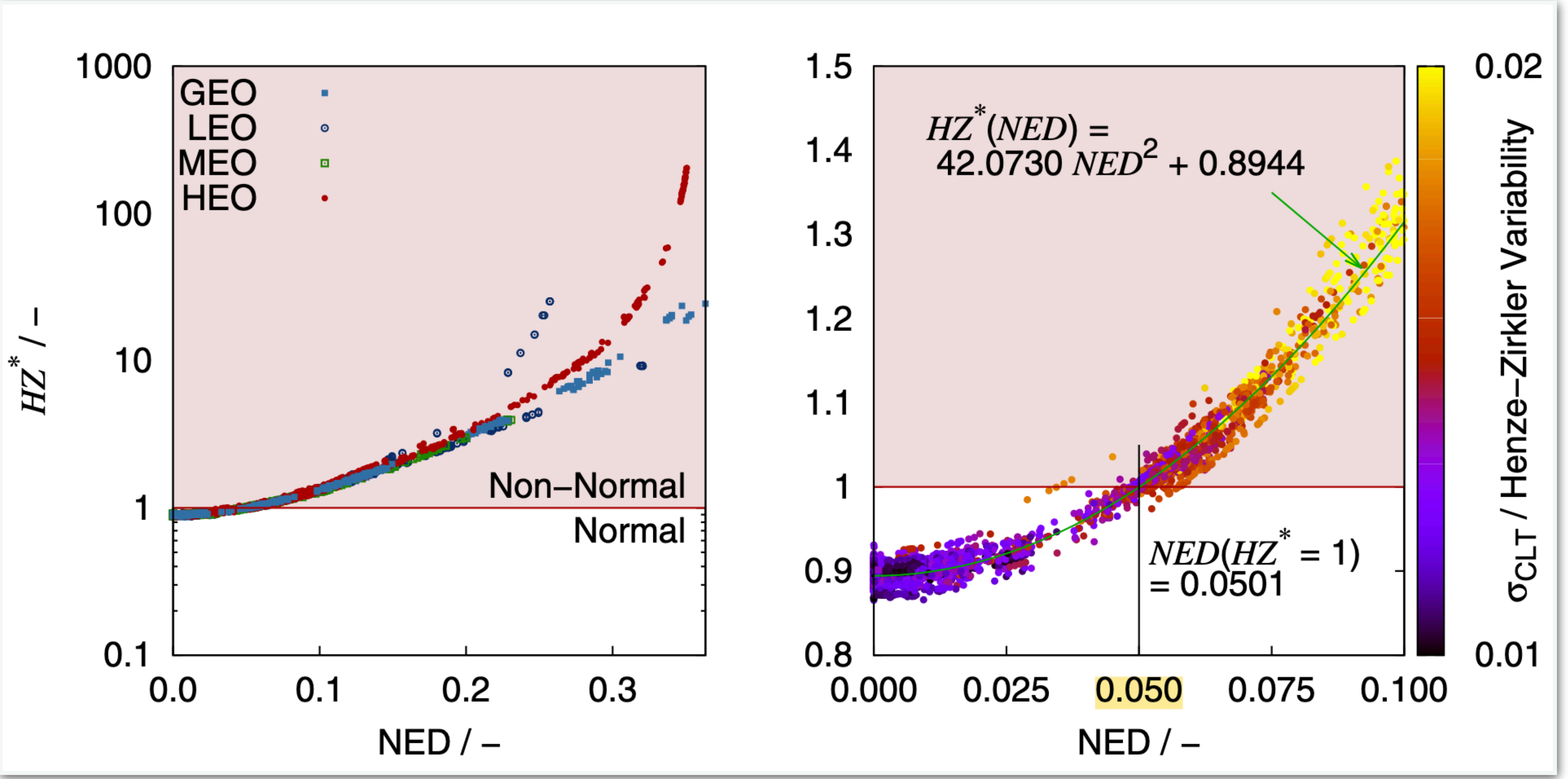
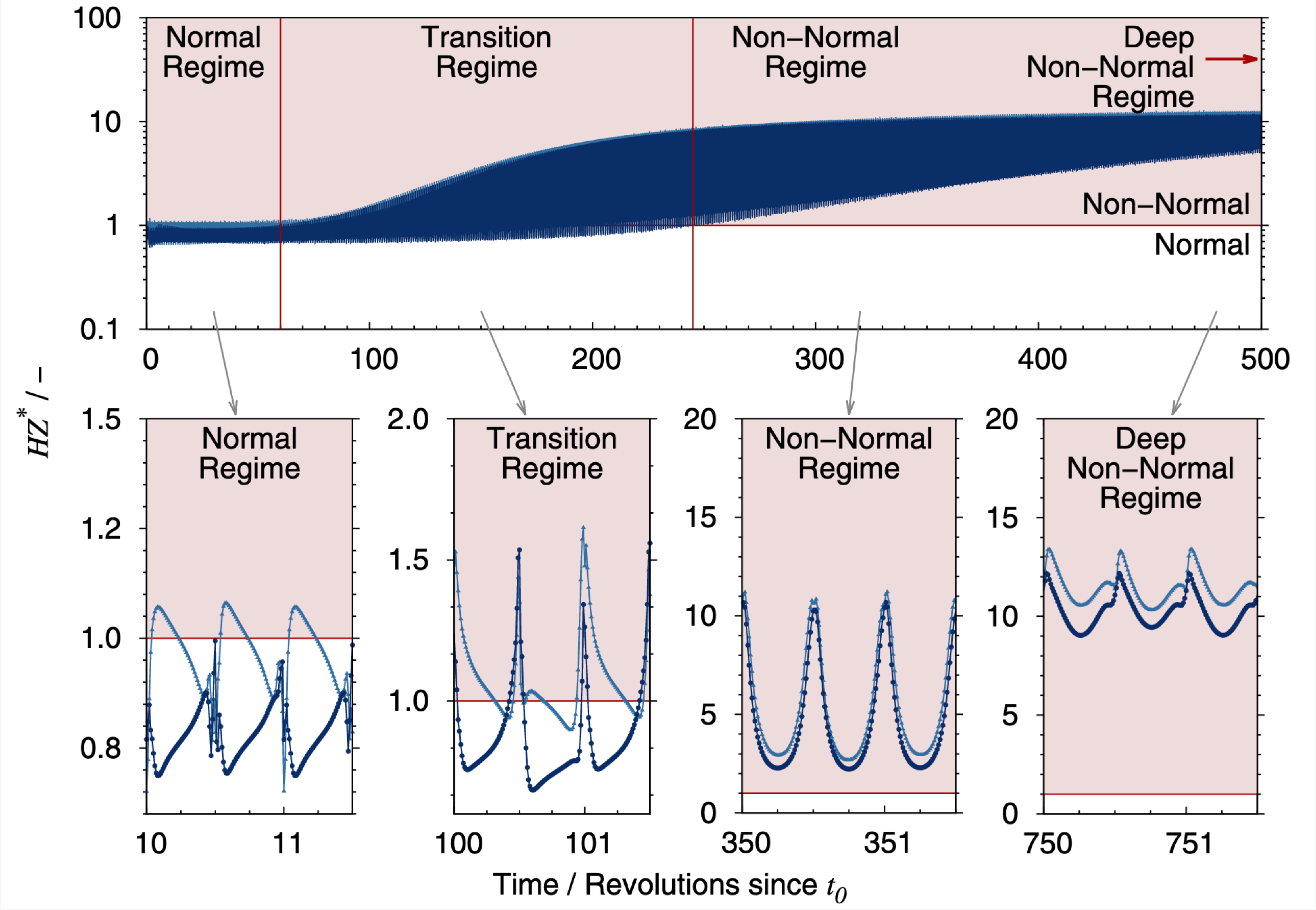




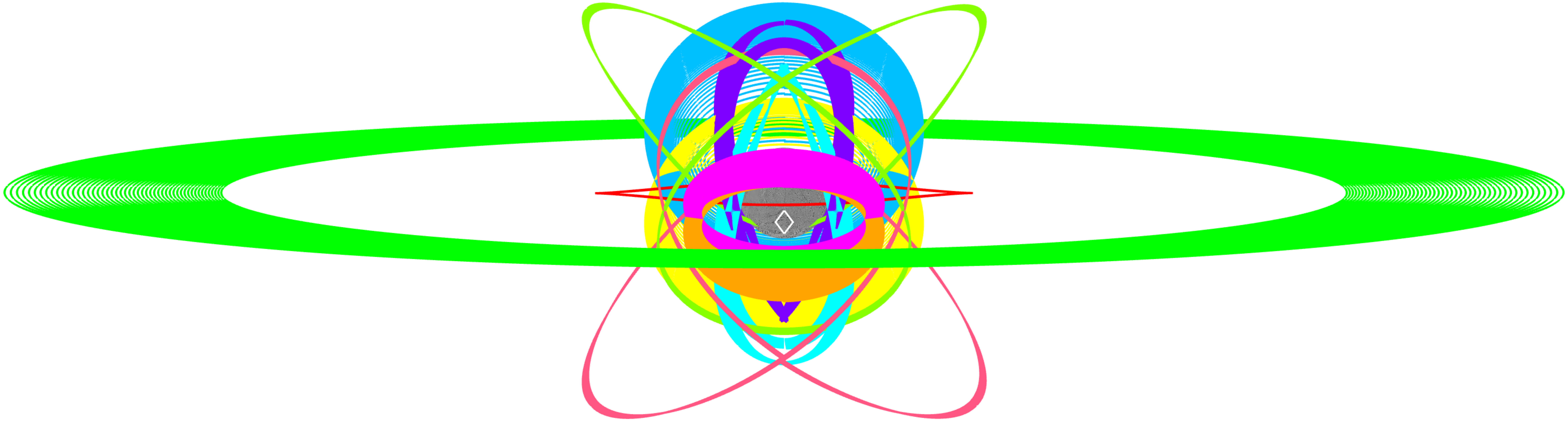
Saturn-Enceladus CR3BP Periodic Orbits

Objective: Determine the relationship (?) between HZ and NED in the Circular Restricted Three-Body Problem using periodic orbit families from the Saturn-Enceladus system.

Using 50 initial conditions from each of the ten following periodic orbit families...

Distant Prograde	Distant Retrograde	L_1 Northern Halo	L_1 Southern Halo	L_2 Northern Halo
L_2 Southern Halo	Northern Butterfly	Northern Dragonfly	Southern Butterfly	Southern Dragonfly

...map the HZ to the NED for the CR3BP.



NASA Jet Propulsion Laboratory, "Three-Body Periodic Orbit Catalog"

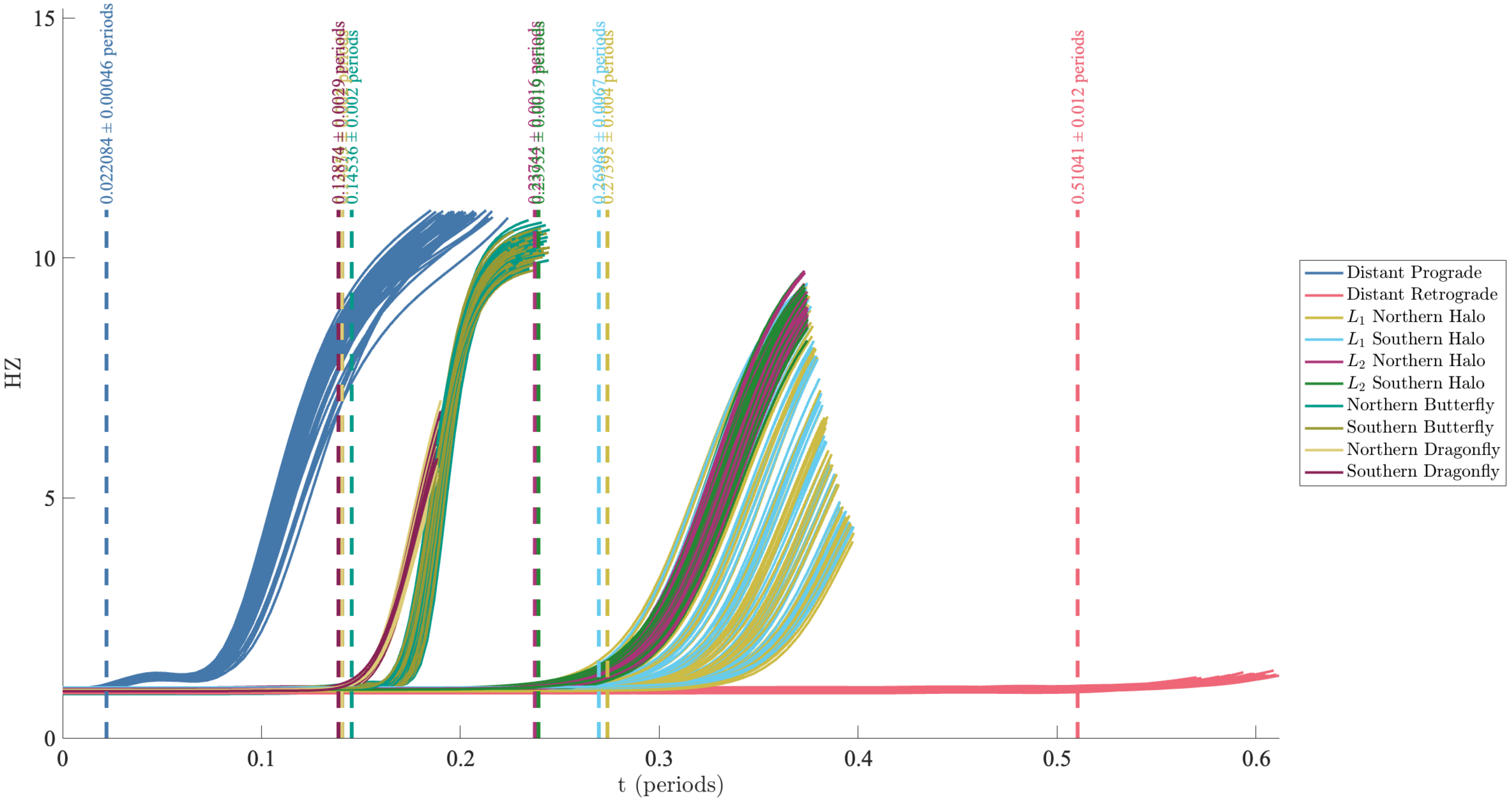
Time to Non-Gaussianity in CR3BP

Saturn-Enceladus Periodic Orbit Families

- Parameters:
  - 50 initial conditions per family, 5,000 random sample size per initial condition,
  - Initial uncertainty: $\sigma_r = 1$ km, $\sigma_v = 1$ cm/s
  - $\mathrm{HZ}^*(\alpha = 0.003) \Rightarrow 3\sigma$ confidence interval

Family	Time to Non-Gaussianity*
Distant Prograde	0.022084±0.00046 periods
Southern Dragonfly	0.134874±0.0029 periods
Northern Dragonfly	0.14033±0.001 periods
Southern Butterfly	0.14059±0.002 periods
Northern Butterfly	0.14536±0.002 periods
L2 Northern Halo	0.23744±0.0016 periods
L2 Southern Halo	0.23932±0.0019 periods
L1 Southern Halo	0.26968±0.0067 periods
L1 Northern Halo	0.27395±0.004 periods
Distant Retrograde	0.51041±0.012 periods

*Mean time ± standard error



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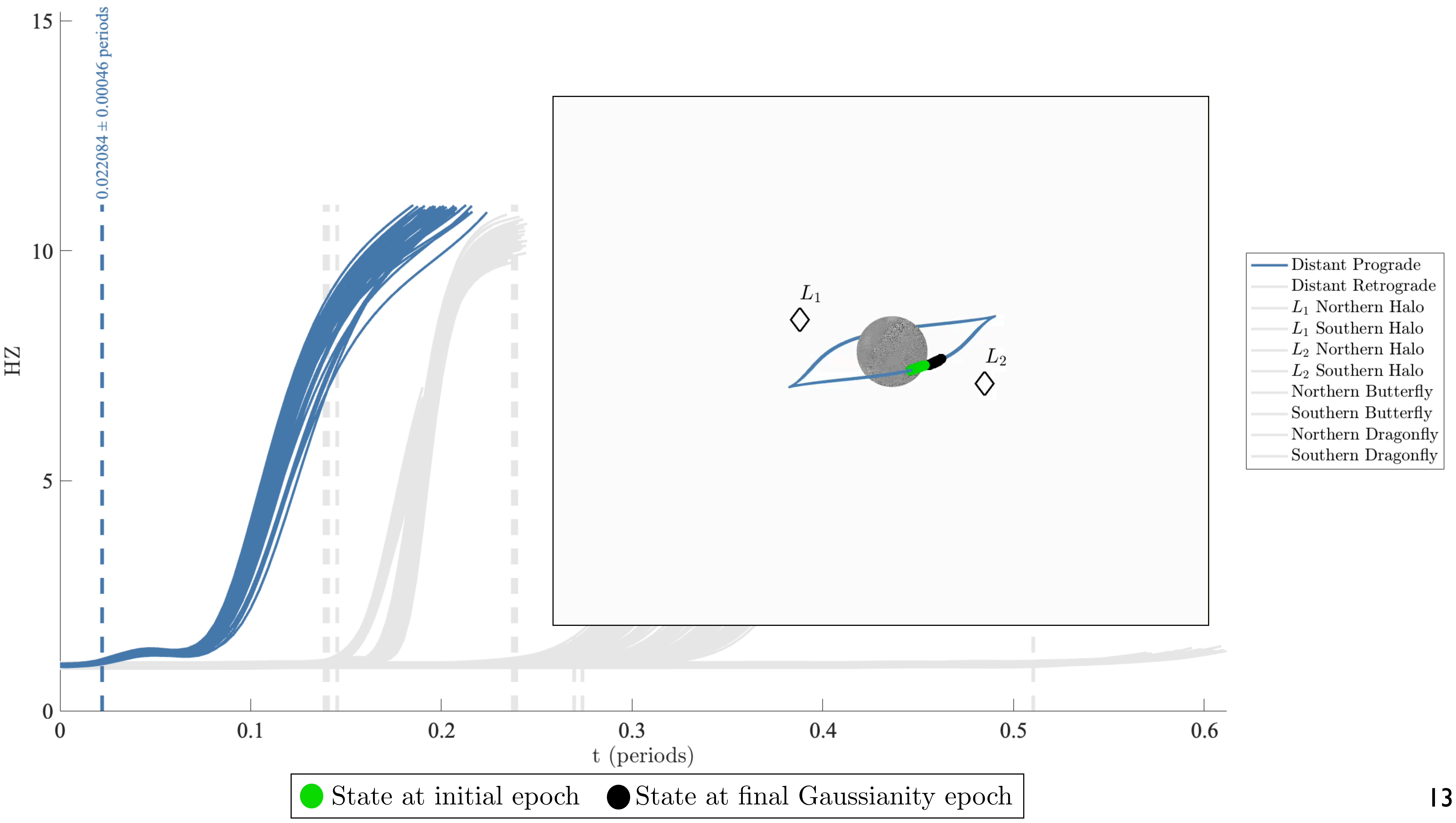

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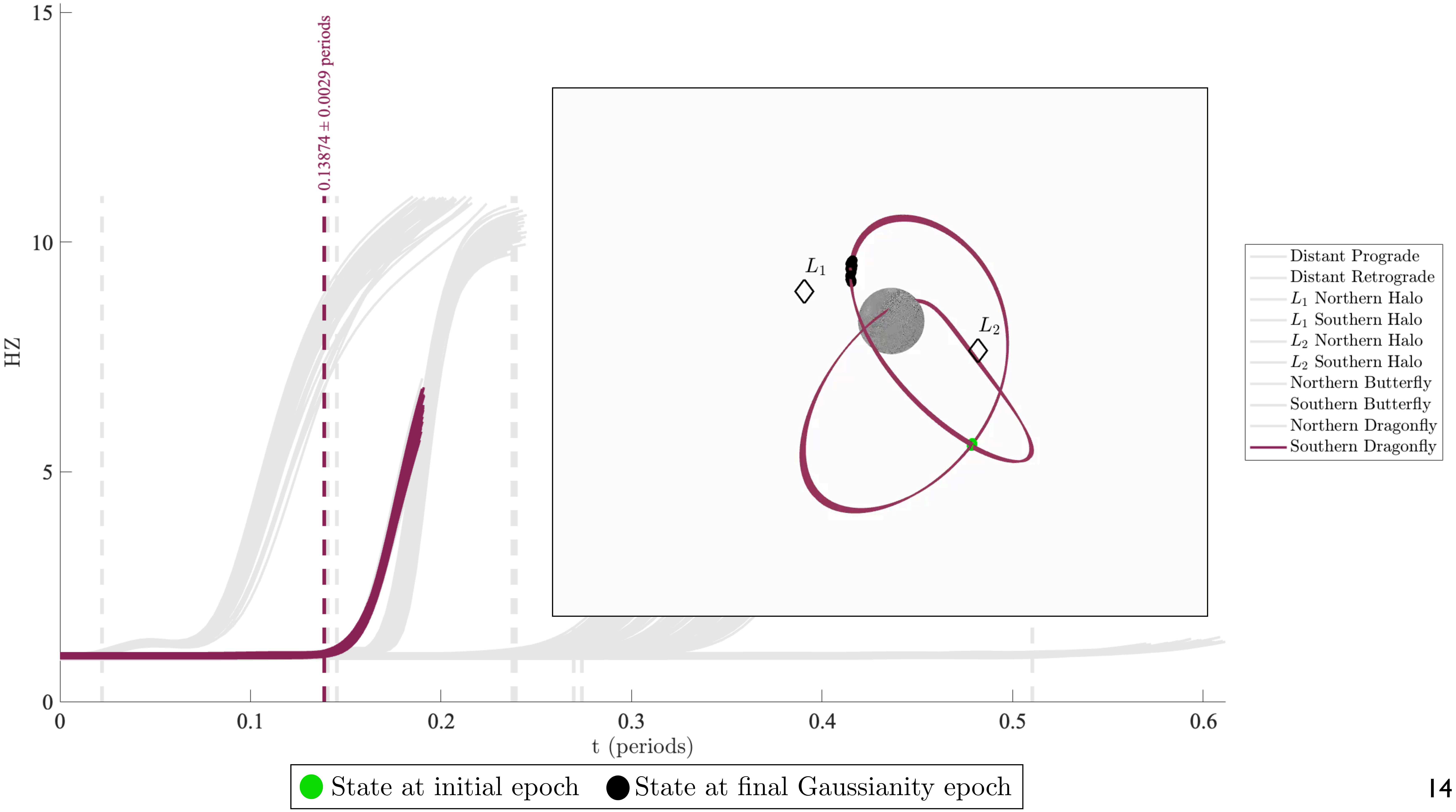

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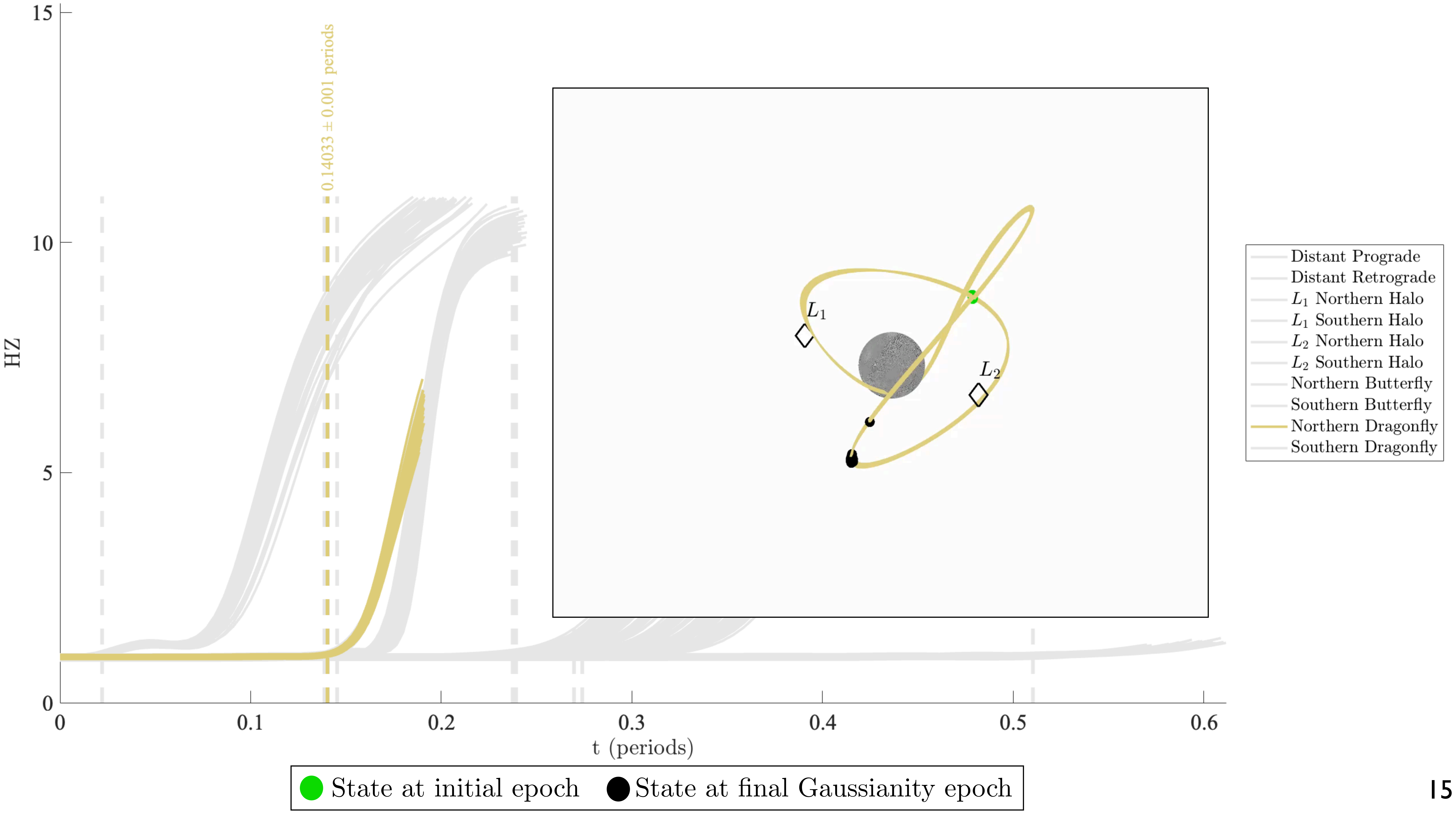

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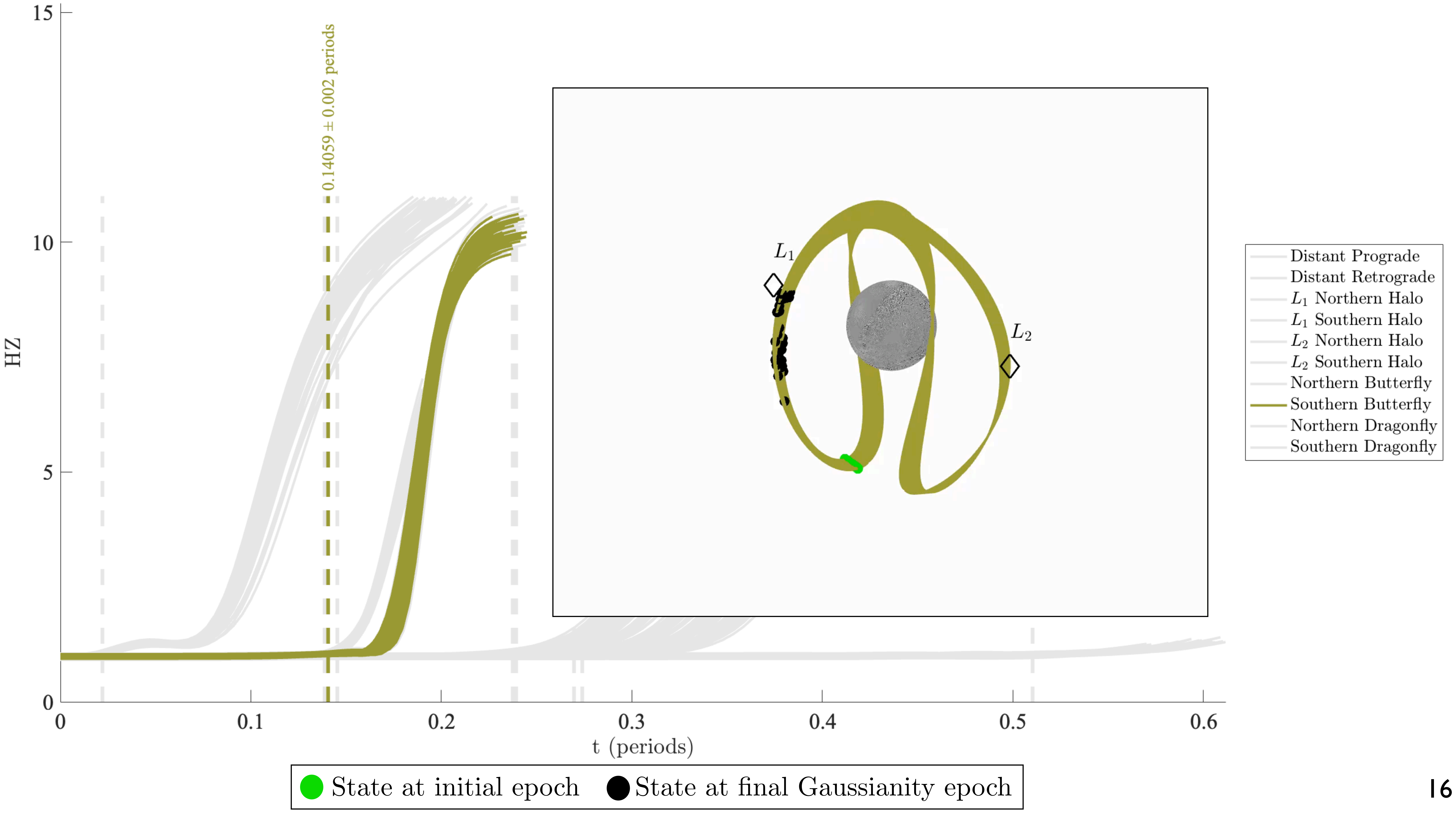

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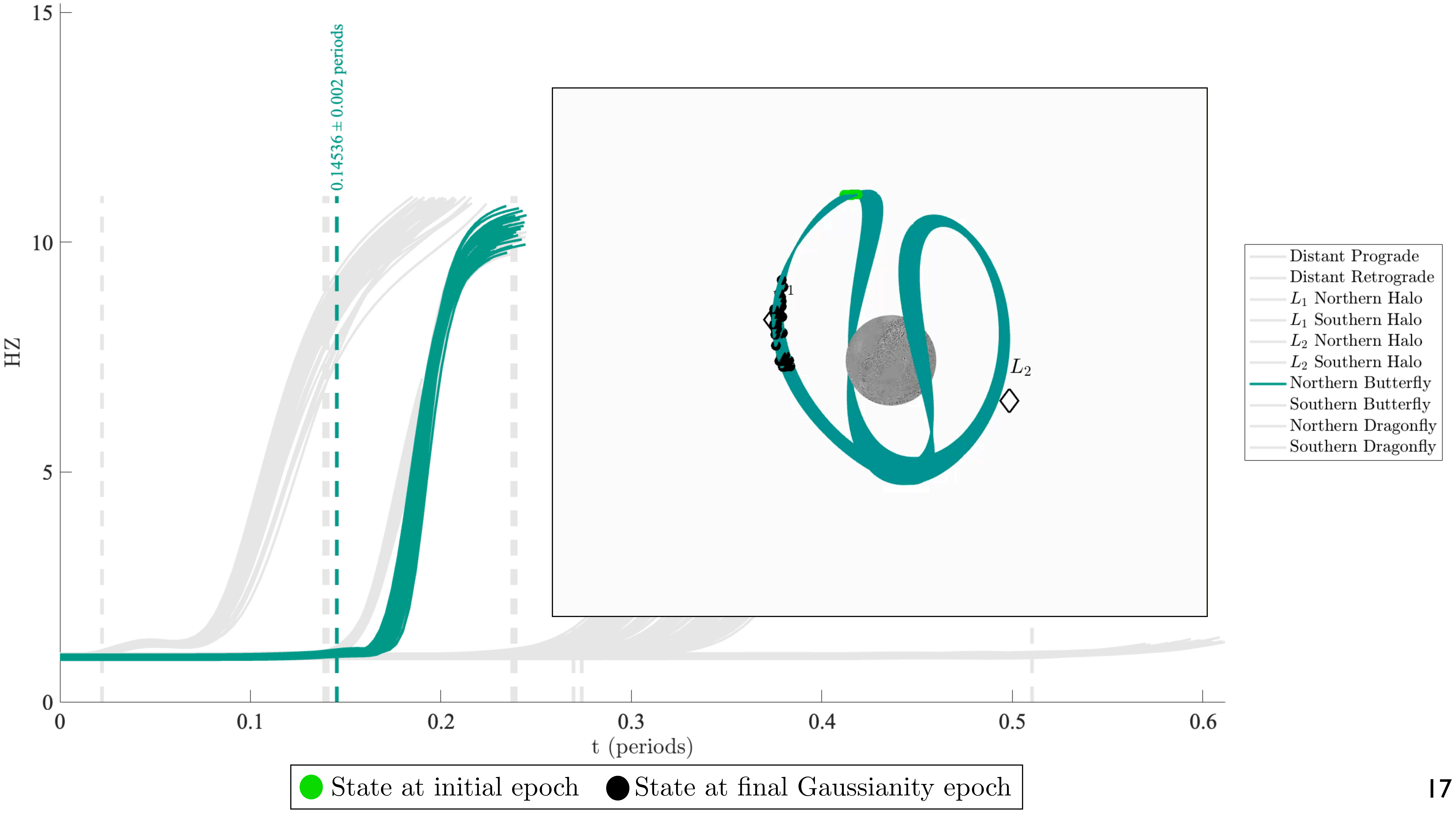

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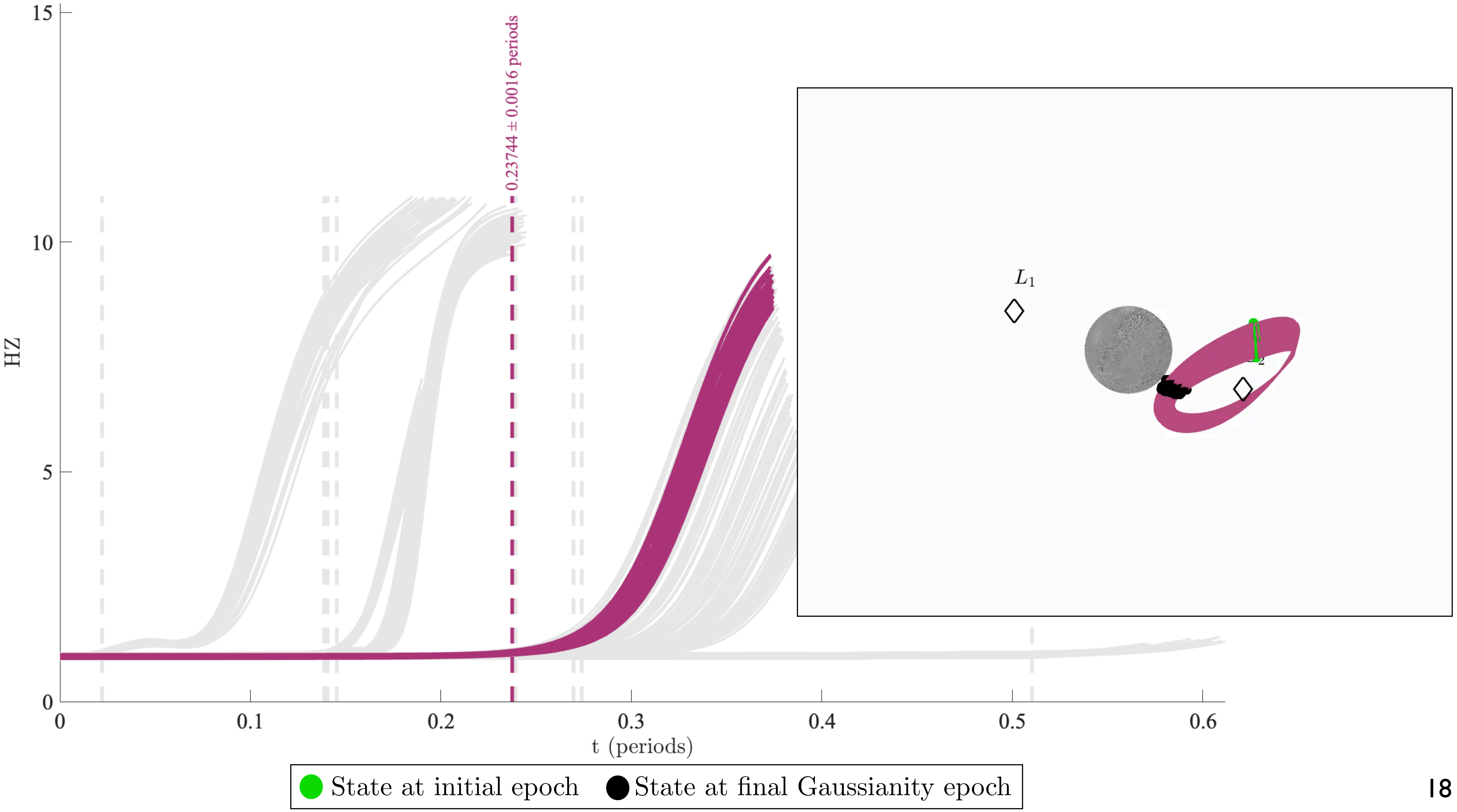

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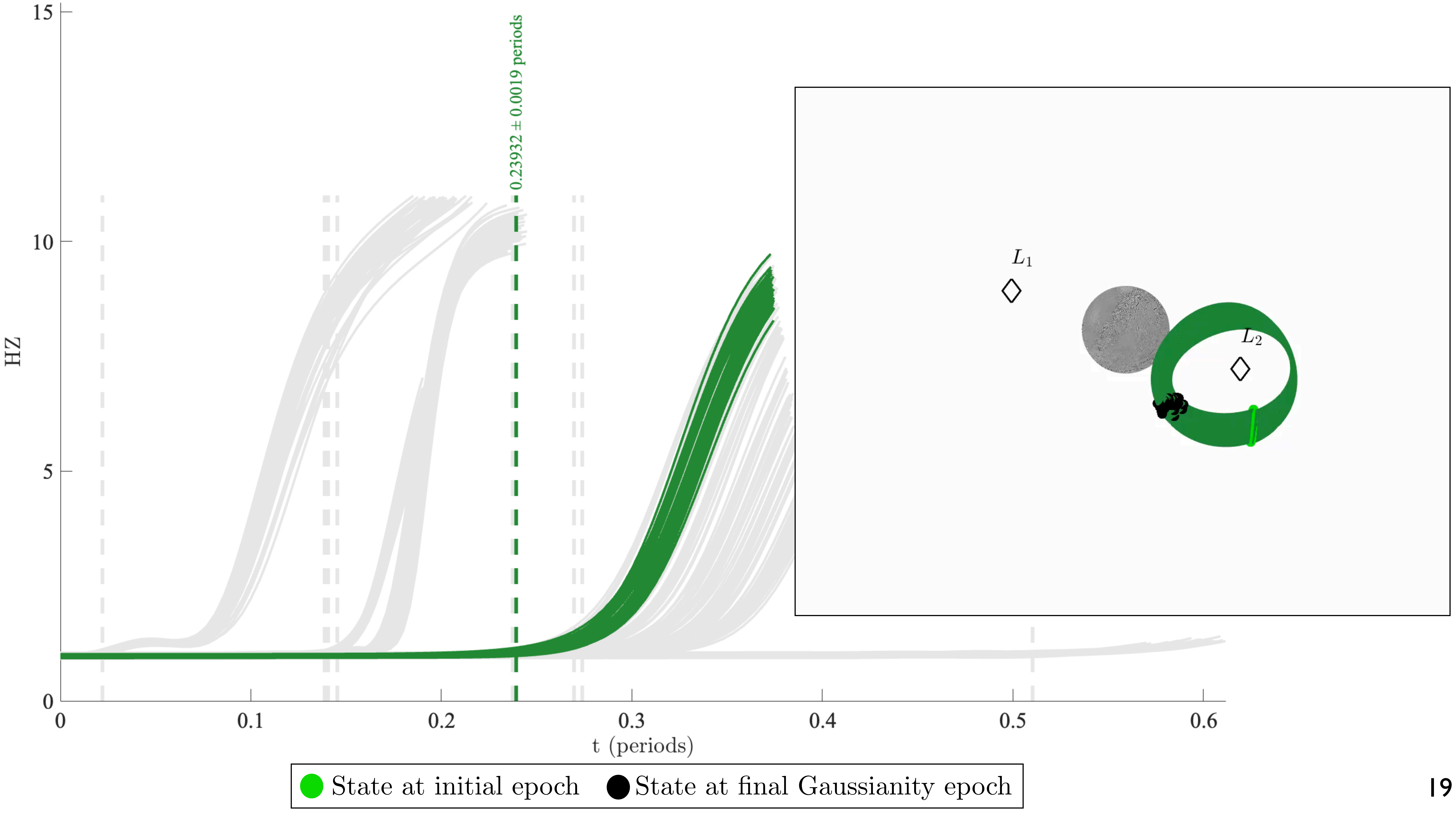

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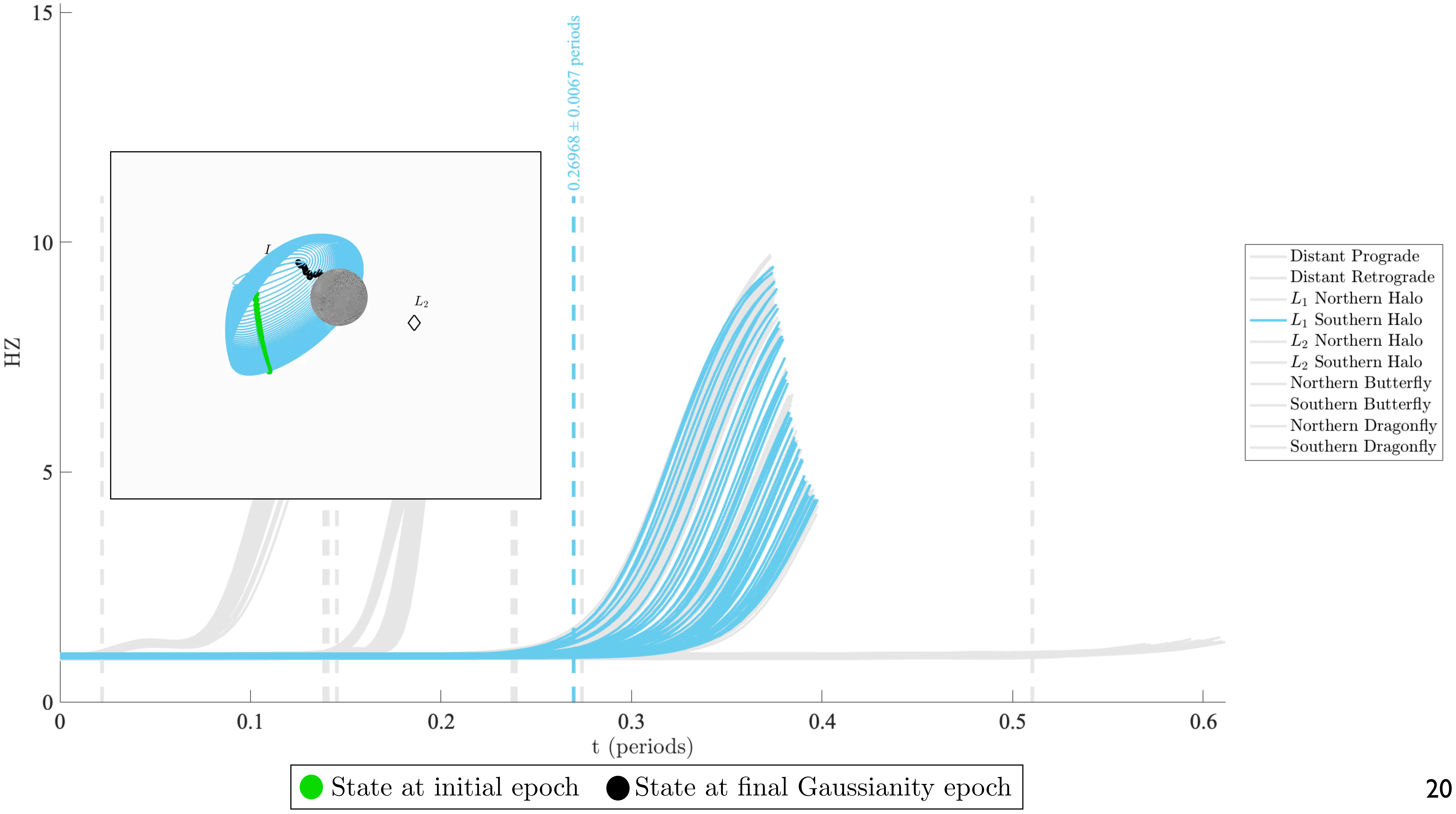

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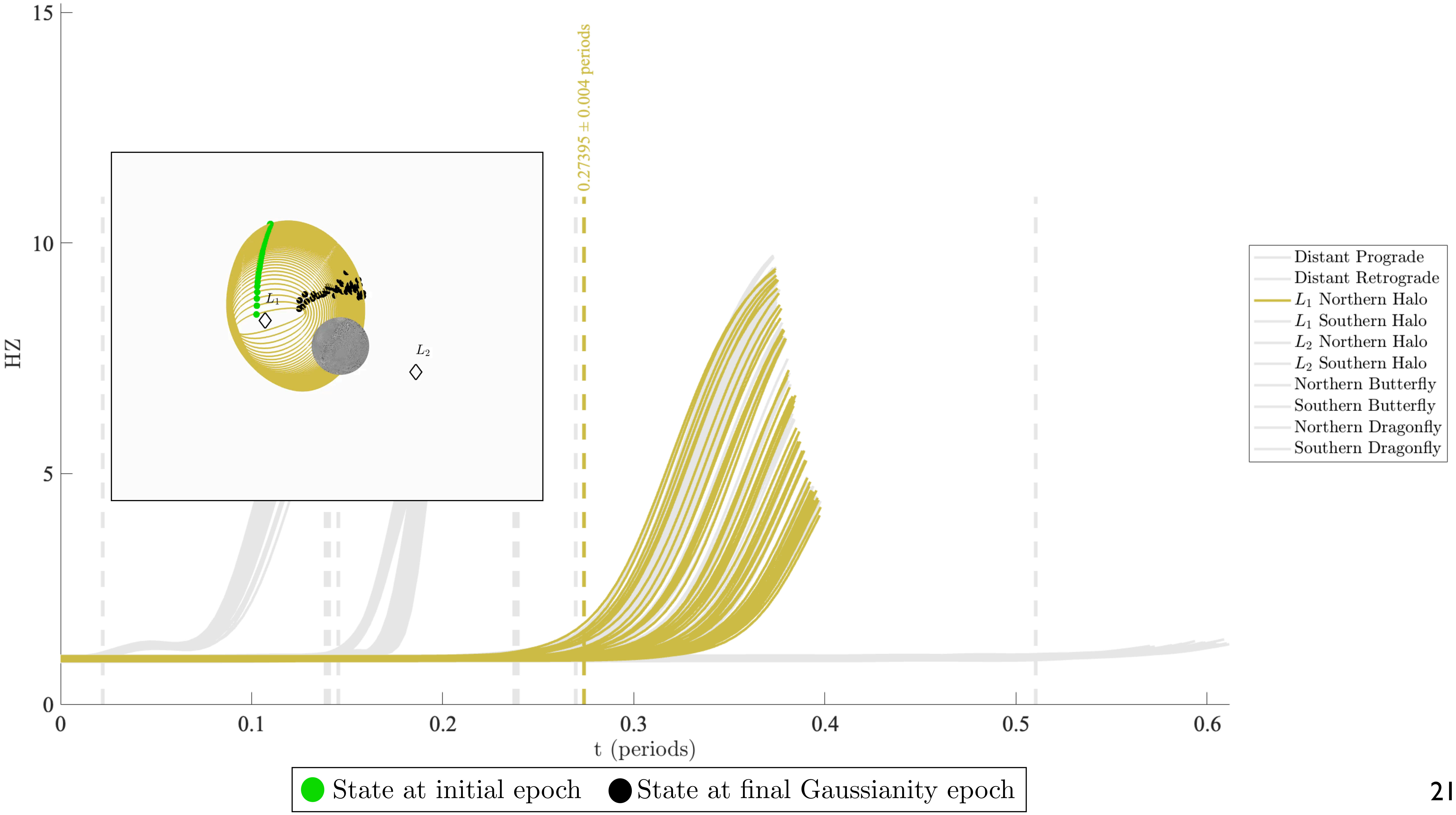

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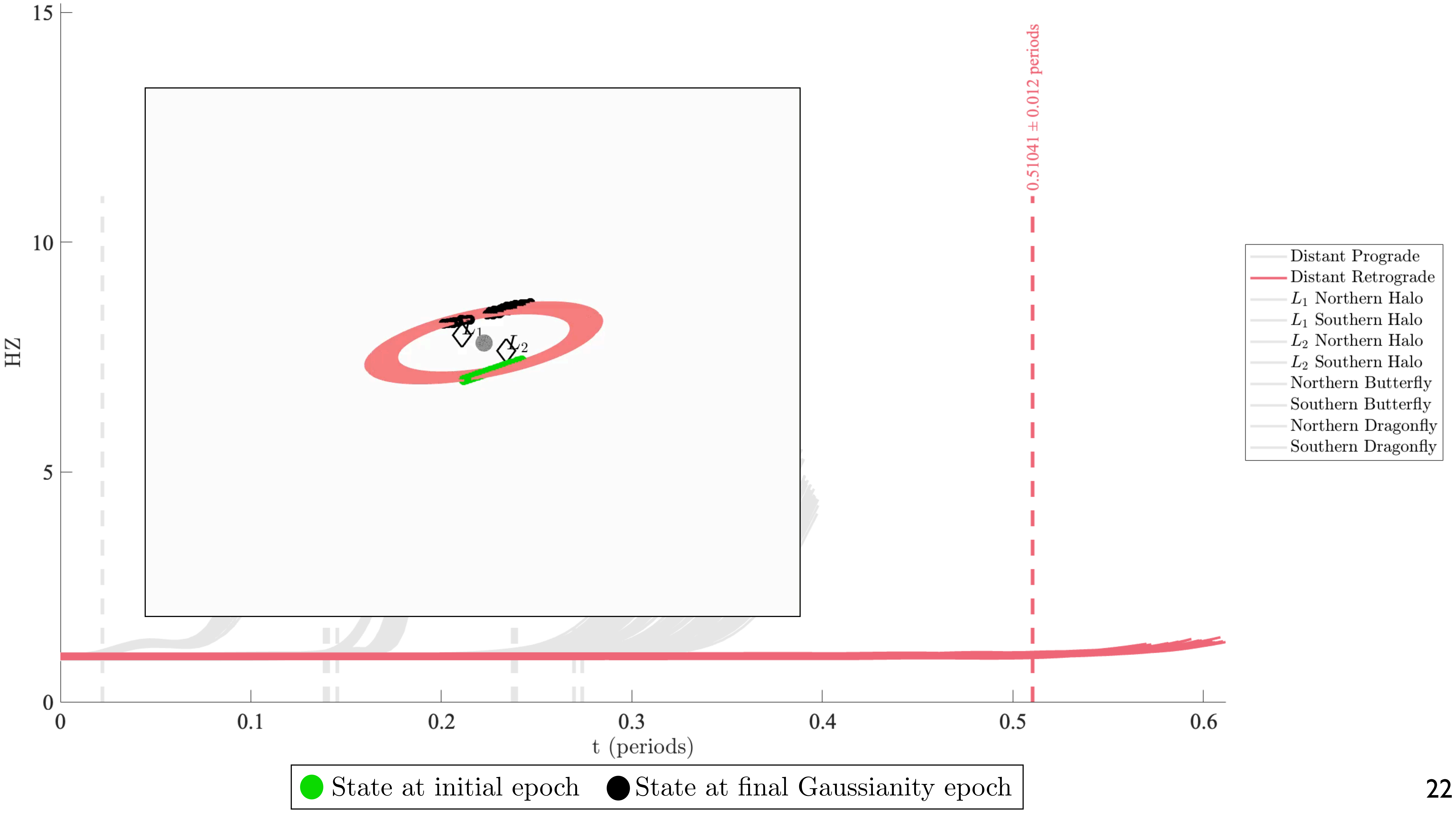
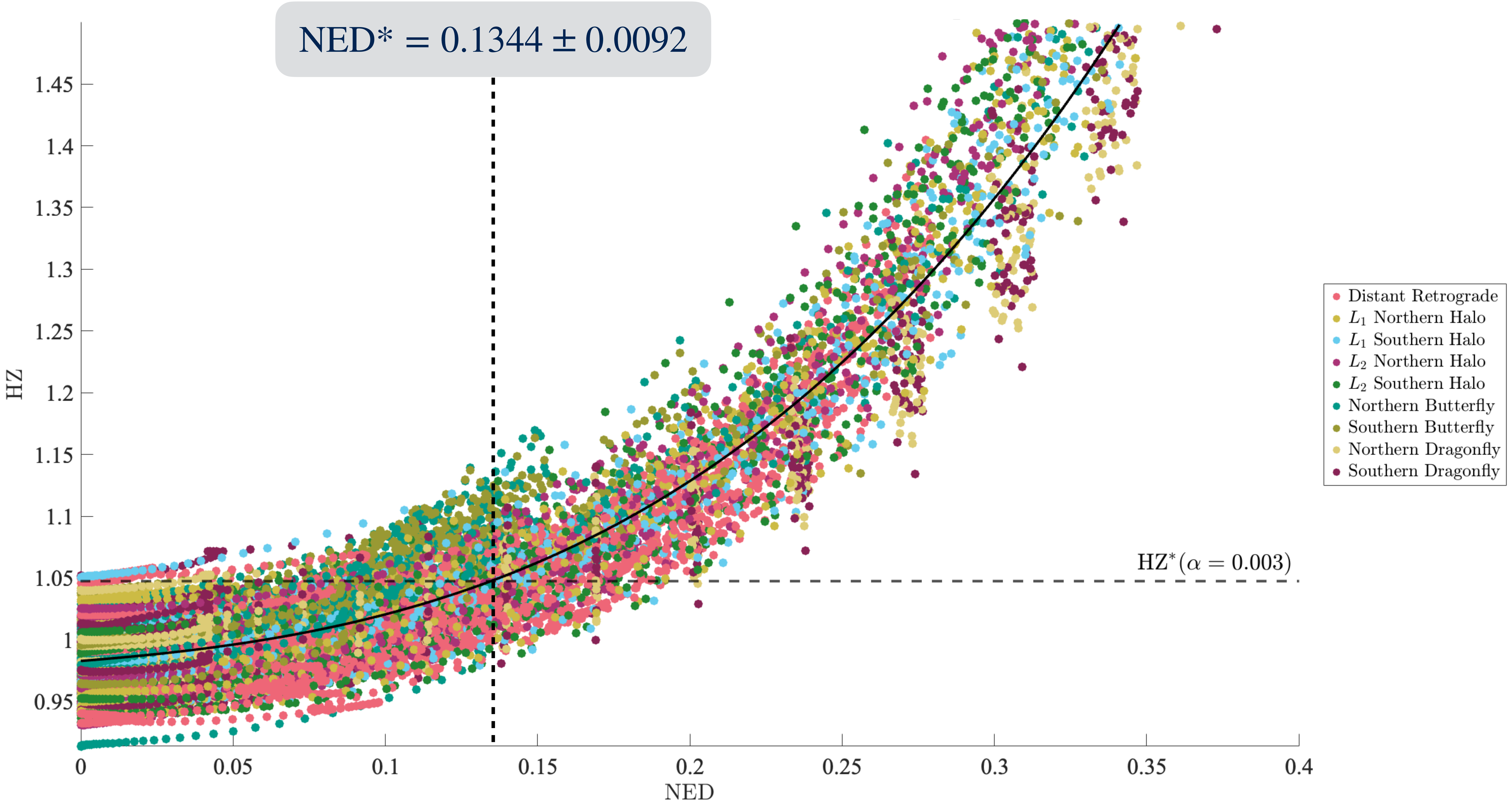


Mapping HZ to NED for Saturn-Enceladus CR3BP

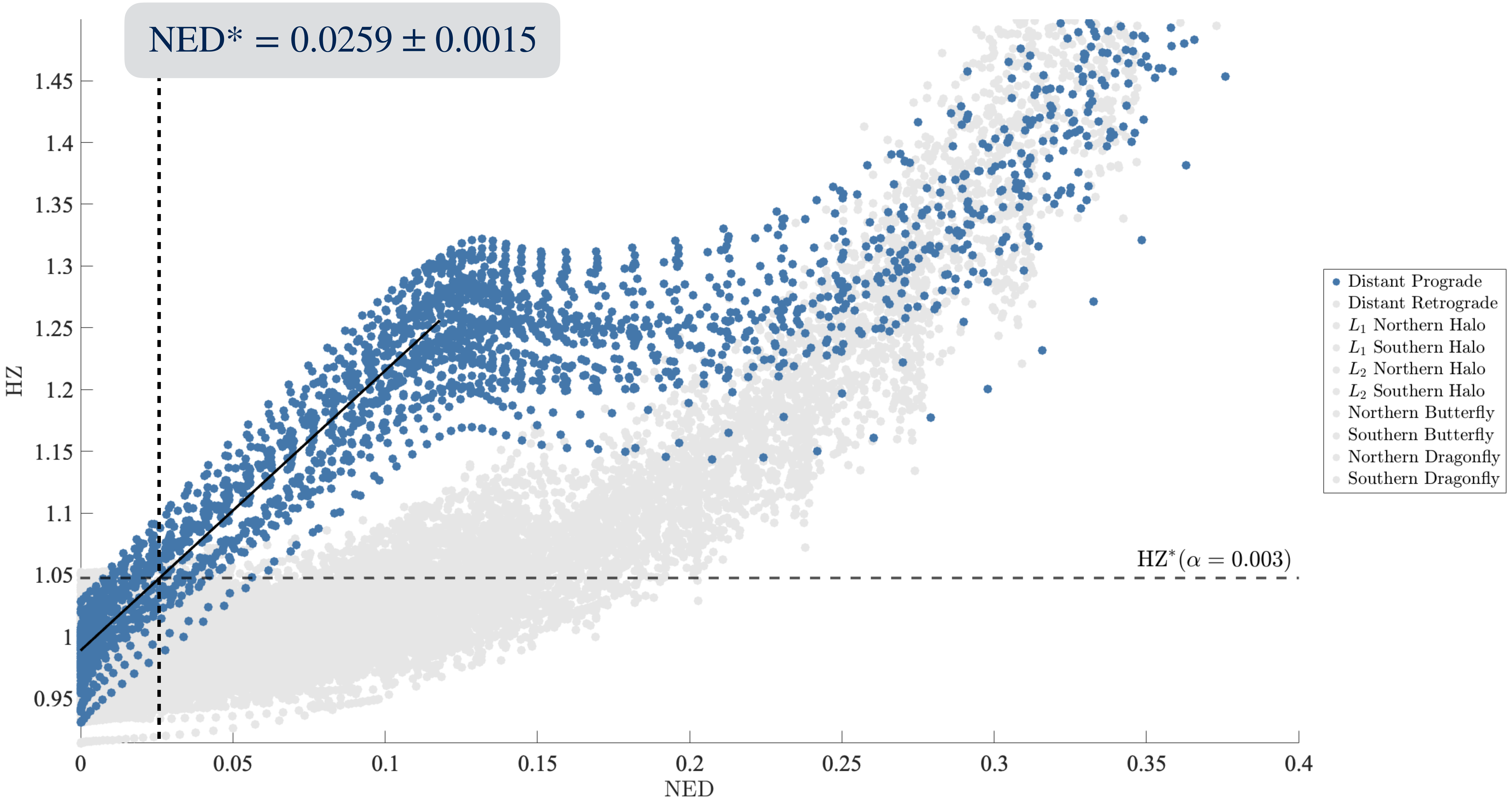


- Curve fit function (3σ confidence intervals):

$$\text{HZ(NED)} = (8.4148 \pm 0.8371)\text{NED}^3 + (0.9872 \pm 0.3490)\text{NED}^2 + (0.1944 \pm 0.0355)\text{NED} + (0.9828 \pm 0.0006)$$

Mapping HZ to NED for Saturn-Enceladus CR3BP

Distant Prograde Orbit Family

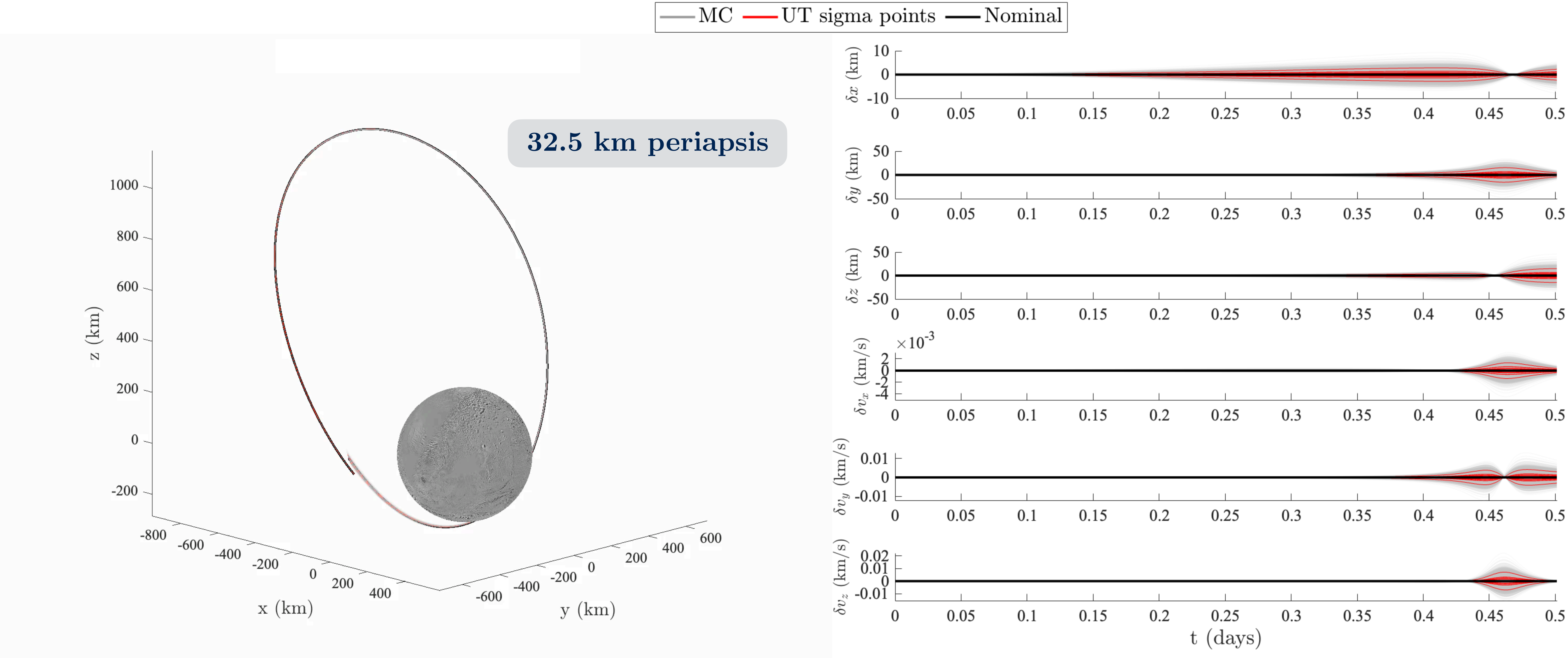


- Curve fit function (3σ confidence intervals):

$$\text{HZ}(\text{NED}) = (2.2654 \pm 0.053202)\text{NED} + (0.98871 \pm 0.0031546)$$

Saturn-Enceladus NRHO Gaussianity Prediction

- We attempt to use our NED* to predict non-Gaussianity applied to a new trajectory



Enceladus NRHO trajectory propagated for 0.5 days, with initial uncertainty $\sigma_r = 100$ m and $\sigma_v = 1$ cm/s.

Russell, R.P. and Lara, M., 2009. On the design of an Enceladus science orbit. Acta Astronautica, 65(1-2), pp.27-39.

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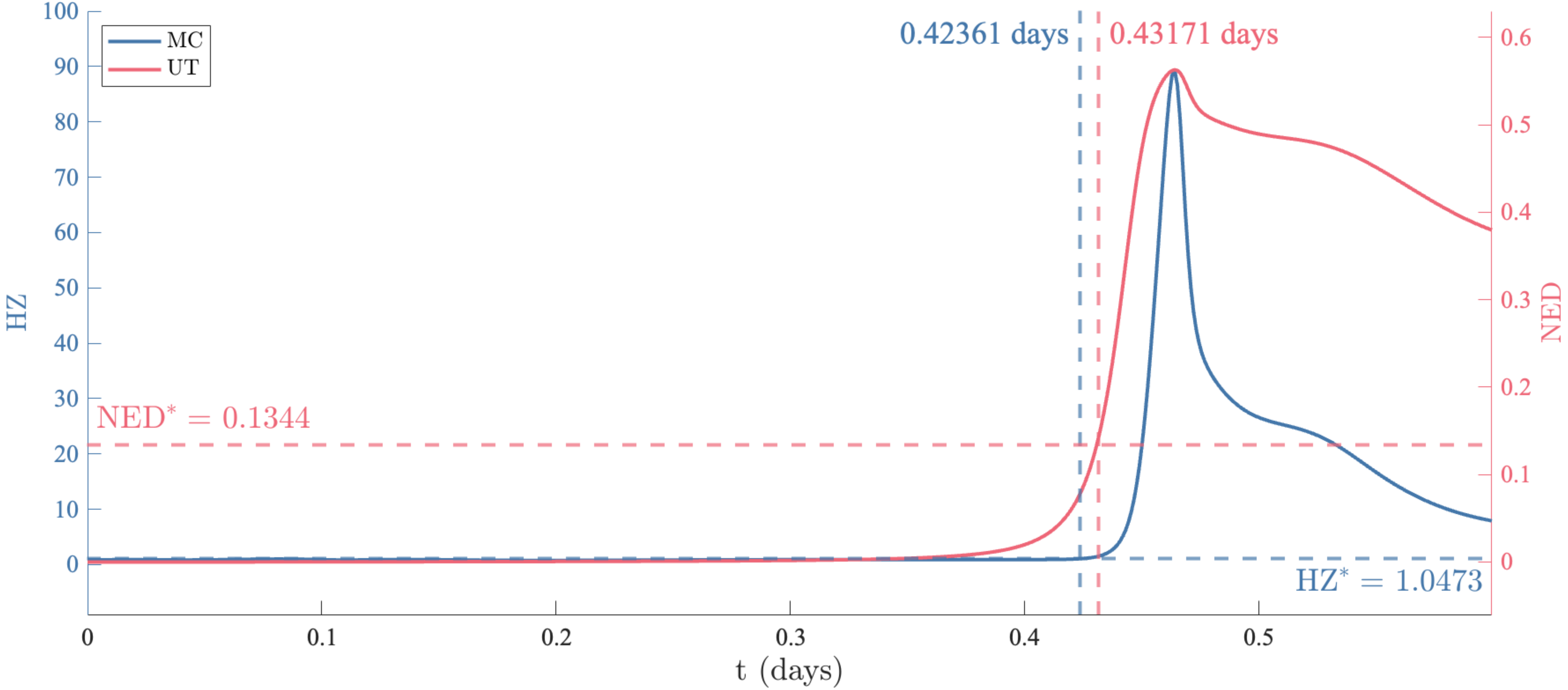


UT is able to predict non-Gaussianity within 12 minutes of a 5,000 sample MC on a completely new trajectory using our derived NED*

Conclusions

Fundamental Questions

1. How do we measure Gaussianity?

$$\text{HZ} = \left[\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\exp\left(-\frac{\beta^2}{2}D_{ij}\right)\right] - \left[2\left(1+\beta^2\right)^{-\frac{d}{2}}\sum_{i=1}^{n}\exp\left(-\frac{\beta^2}{2(1+\beta^2)}D_i\right)\right] + \left[n(1+2\beta^2)^{-\frac{d}{2}}\right]$$

2. How long does it take for state uncertainty to become non-Gaussian ($\sigma_r = 1$ km, $\sigma_v = 1$ cm/s)?

Family	Distant Prograde	Southern Dragonfly	Northern Dragonfly	Southern Butterfly	Northern Butterfly	L2 Northern Halo	L2 Southern Halo	L1 Southern Halo	L1 Northern Halo	Distant Retrograde
***t* (periods)**	0.022084	0.134874	0.14033	0.14059	0.14536	0.23744	0.23932	0.26968	0.27395	0.51041

3. Can we predict when state uncertainty is becoming non-Gaussian with an abstraction more efficient to propagate than a dense Monte Carlo?

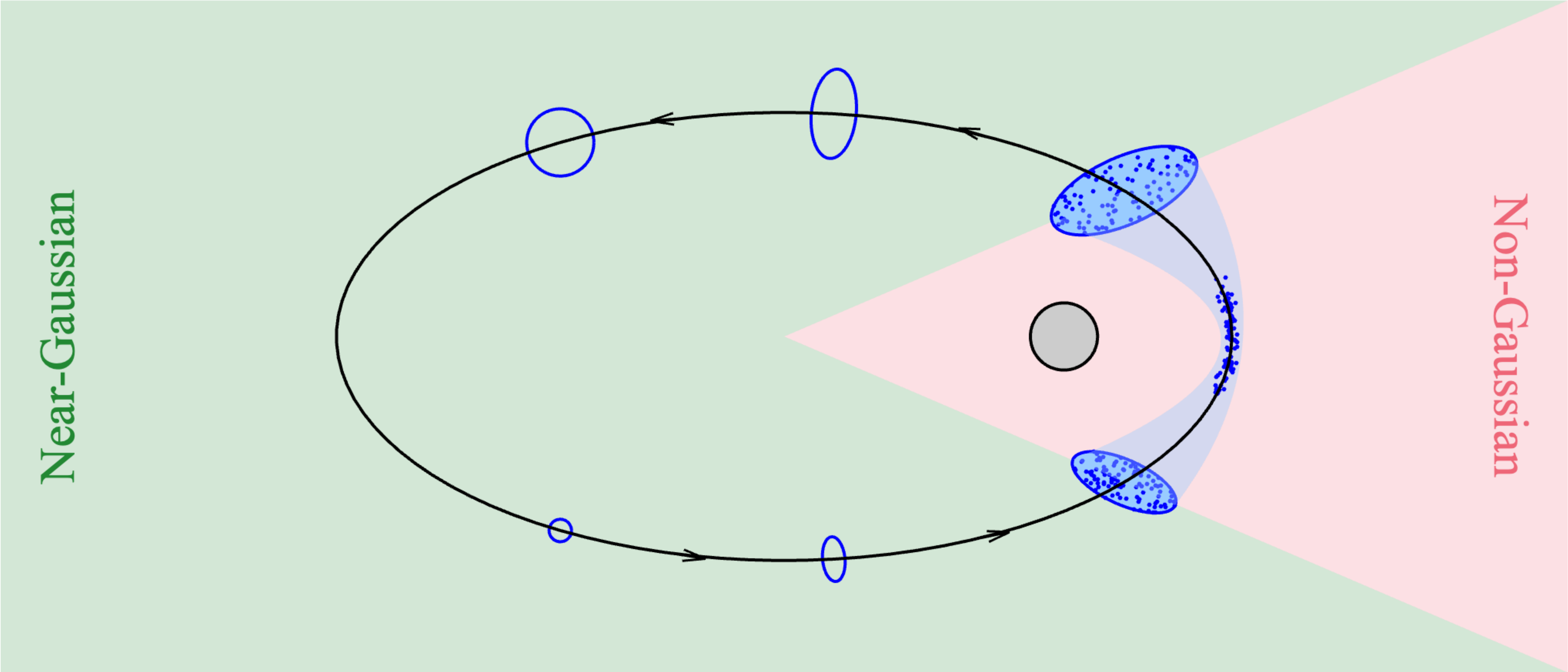
- Using 500 different periodic orbits from the Saturn-Enceladus system, we successfully mapped the NED to the HZ for the CR3BP

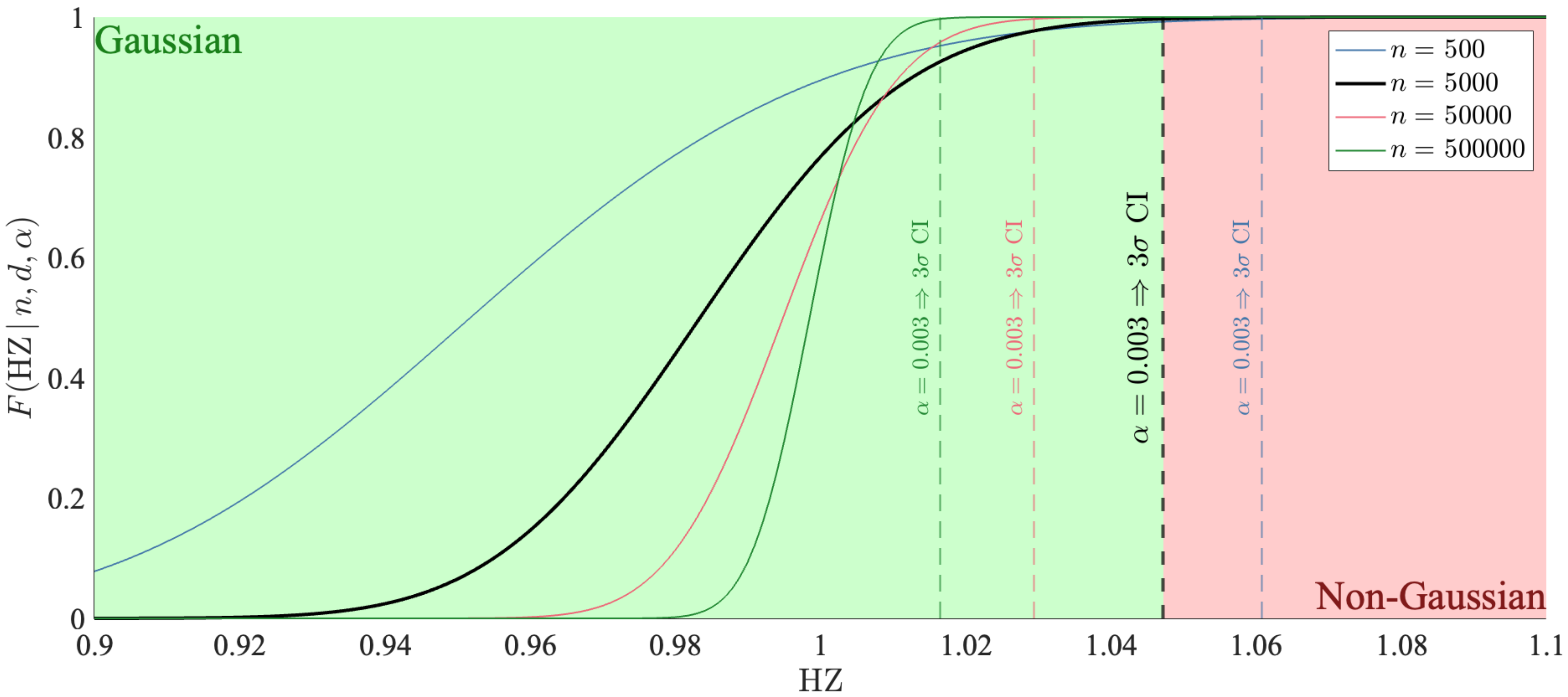
$$\text{NED}^* = 0.1344 \pm 0.0092$$

Future Work

Hybrid Filtering

- Using the NED* value derived in this work, we can develop a hybrid filter that propagates the first and second moments when uncertainty is near-Gaussian, and an ensemble distribution when the uncertainty is non-Gaussian
- Hybrid filter would be more accurate than a pure moment filter and more efficient than a pure ensemble filter





Sparse MC Gaussianity Detection

- NED must be mapped for each uncertainty magnitude and dynamics model, while HZ is a consistent statistic no matter the model or uncertainty
- What are the Type I/II error rates for a sparse MC distribution compared for the large one used in this analysis?

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Thanks to Dr. Ely and Dr. Lo for mentoring and co-mentoring this summer, as well as for their invaluable insight and contributions.

Back of the envelope calculation:

$$10\,\text{families} \times 50\frac{\text{orbits}}{\text{family}} \times 5{,}000\frac{\text{sample size}}{\text{orbit}} \times 20\frac{\text{mistakes}}{\text{sample size}} = \textbf{50 million trajectories propagated!}$$

This would not have been possible without the Monte parallelization module, thanks to Margaret Ryback for helping me get this set up!

Thank you for your time. Questions?